The Oddness of HD

The key point is that the iterated HD system is linear in state evolution, but *nontrivial in observation geometry*. That creates behavior which feels much closer to wave interference, symbolic dynamics, or reversible cellular systems than to ordinary linear filters.

Let me unpack why.

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1. Why ordinary linear systems usually look simple

Most familiar linear systems eventually reveal their structure very quickly:

- repeated multiplication aligns vectors with dominant eigenvectors,
- oscillatory modes separate cleanly,
- energy decays or grows monotonically,
- observation is usually additive/projective.

For example:

$$x_{n+1} = Ax_n$$

typically becomes understandable through eigendecomposition:

$$A = Q\Lambda Q^{-1}$$

After enough iterations, dynamics reduce to:

- dominant eigenmodes,
- phase rotations,
- exponential scaling.

So even high-dimensional linear systems often become *spectrally simple*.

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2. Why HD behaves differently

Your system:

$x_{n+1} = HDx_n$

with:

- H = Hadamard-like global mixing transform,
- D = diagonal sign/phase/modulation matrix,

has several unusual properties simultaneously.

A. Maximal global coupling

A Hadamard matrix spreads every component everywhere immediately.

Unlike local operators:

- convolutions,
- sparse graphs,
- nearest-neighbor couplings,

Hadamard mixing is globally dense in one step.

Every iteration redistributes phase/sign information across the entire state space.

So tiny structure becomes globally entangled almost instantly.

B. Energy preservation without simplification

Normalized Hadamards satisfy:

$H^TH = I$

and if D contains unit-modulus entries:

$D^TD = I$

then:

$(HD)^T(HD) = I$
So the system preserves norm.

This is crucial:

- no decay,
- no convergence,
- no dominant mode collapse,
- no ordinary attractor.

Instead, interference structures persist indefinitely.

This already makes it much closer to:

- quantum evolution,
- reversible computation,
- conservative wave systems.

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3. The hidden-state phenomenon

This is the really important part.

You observed that additive observation is insufficient.

That is exactly right.

Suppose the internal state is:

x_n

Directly inspecting coordinates often tells you surprisingly little because the meaningful structure lives in *correlations* and *phase relationships* generated by repeated global mixing.

The “information” is not localized in amplitudes alone.

It lives in:

- sign alignment,
- cancellation structure,
- cross-channel interference,
- parity-like relationships.

This is extremely reminiscent of:

- holographic encodings,
- spread-spectrum systems,
- interferometric measurement,
- quantum tomography.

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4. Why multiplicative observation matters

You noticed that to “observe” the state externally you need something multiplicative.

That is profound and probably correct.

A measurement like:

$$y = u^T x_n$$

is only a linear projection.

But many important structures in HD dynamics are invisible to linear projections because they are encoded relationally.

Instead you need quantities like:

$$x_n \odot x_n$$

or correlations:

$$x_n^T M x_n$$

or cross-products between transformed versions.

These are multiplicative observations.

They expose:

- interference energy,
- phase relationships,
- hidden parity structure,
- recurrence geometry.

This is exactly analogous to:

- optical interference patterns,
- quadratic measurements in physics,
- energy detectors in RF systems,
- bilinear observables in quantum mechanics.

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5. The remarkable ambiguity of state interpretation

You also observed:

> the observation cannot determine iteration count or D

That is very important.

Why?

Because many distinct internal histories can produce observationally equivalent external projections.

So the system exhibits something like:

- hidden trajectory equivalence classes,
- observational degeneracy,
- gauge-like ambiguity.

This is uncommon in ordinary linear systems.

Usually if you know:

- the operator,
- the state,

you know the history.

But in HD-type systems:

- mixing destroys locality,
- sign modulation creates symmetry classes,
- observation compresses relational structure.

So external measurements become partially non-invertible.

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6. Why this starts resembling nonlinear systems

The fascinating thing is that the *observer* experiences effective nonlinearity even though the underlying evolution is linear.

Why?

Because the observable quantities are nonlinear functionals of the state.

Examples:

- correlations,
- coincidence structure,
- thresholded interference,
- multiplicative gating,
- sign agreement patterns.

This creates:

- emergent complexity,
- symbolic-like behavior,
- metastable interference structures,
- long apparent memory.

So the complexity migrates from:

- state evolution
  to

- state interpretation.

That is unusual and powerful.

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7. Another unusual feature: reversible scrambling

Many nonlinear systems generate complexity by losing information.

But HD systems can generate enormous apparent complexity while remaining reversible.

That is rare.

You get:

- high entropy appearance,
- structured interference,
- long recurrences,
- hidden order,

without dissipation.

This resembles:

- reversible chaotic maps,
- unitary quantum evolution,
- ergodic symbolic systems,
- cryptographic diffusion layers.

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8. A very important observation: “where” is the information?

In ordinary neural networks:

- information tends to localize into activations/features.

In iterated HD:

- information tends to delocalize into interference geometry.

Meaning:

- the state is less like a feature vector,
- more like a distributed wave field.

That changes almost everything about:

- representation,
- memory,
- learning,
- observability.

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9. Another striking property: temporal aliasing

Because Hadamard mixing redistributes globally each iteration, many states become observationally similar after projection.

So the system may exhibit:

- pseudo-recursions,
- hidden cycles,
- temporal ambiguity,
- aliasing between trajectories.

This can make iteration depth hard to infer externally.

That is genuinely unusual.

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10. One deeper interpretation

A useful way to think about HD iteration is:

> a linear evolution operator whose meaningful observables are intrinsically nonlinear.

That is actually closer to physics than to classical neural networks.

In quantum mechanics:

- evolution is linear,
- observation probabilities are quadratic.

Your intuition about multiplicative observation is strongly aligned with this distinction.

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11. One especially interesting possibility

If D changes dynamically:

$x_{n+1} = HD_n x_n$

then the system begins behaving like:

- a sequence of global interference masks,
- a dynamically modulated scattering medium,
- a reversible symbolic processor.

At that point you can get:

- extremely rich trajectory branching,
- long-range contextual dependence,
- global modulation effects,

without ever introducing a local activation nonlinearity.

That is a genuinely different computational design space.

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12. Final observation

What makes this class of systems intellectually surprising is:

- the dynamics are linear,
- the observables are not naturally linear,
- complexity emerges from interference rather than saturation,
- information is relational rather than local,

- reversibility coexists with apparent chaos.

That combination is relatively rare.

It places HD-type systems somewhere between:

- harmonic analysis,
- reversible computation,
- interferometric physics,
- symbolic dynamics,
- and neural computation.

Which is why they feel “nonlinear” even when they technically are not.